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M Com –II Sem. IV - Study Material

Subject: Quantitative Technique

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Unit-II - TRANSPIRATION PROBLEM

INTRODUCTION

There are two phases to solve the transportation problem. In the first phase, the initial basic feasible solution has to be found and the second phase involves optimization of the initial basic feasible solution that was obtained in the first phase. **There are three methods for finding an initial basic feasible solution,**

I - Northwest Corner Method.(NCM) - <https://youtu.be/ItOuvM2KmD4>

II - Least Cost Cell Method. (LCM) - Video - <https://youtu.be/5xCUzCTjAXQ>
<https://youtu.be/K4D1Z1c4cwA>

III- Vogel's Approximation Method.(VAM) https://youtu.be/dbhBTGTZ_WU
https://youtu.be/ip6Hu_jmMeY
<https://youtu.be/AjvKrXEKIoc>

I- NORTH-WEST CORNER METHOD (NWC)

- i. Select the North-west (i.e., upper left) corner cell of the table and allocate the maximum possible units between the supply and demand requirements. During allocation, the transportation cost is completely discarded (not taken into consideration).
- ii. Delete that row or column which has no values (fully exhausted) for supply or demand.
- iii. Now, with the new reduced table, again select the North-west corner cell and allocate the available values.
- iv. Repeat steps (ii) and (iii) until all the supply and demand values are zero.
- v. Obtain the initial basic feasible solution.

II LEAST COST METHOD (LCM)

- i. Select the smallest transportation cost cell available in the entire table and allocate the supply and demand.
- ii. Delete that row/column which has exhausted. The deleted row/column must not be considered for further allocation.
- iii. Again select the smallest cost cell in the existing table and allocate. (Note: In case, if there are more than one smallest costs, select the cells where maximum allocation can be made)

- iv. Obtain the initial basic feasible solution.

III- VOGEL'S APPROXIMATION METHOD (VAM)

- i. Calculate penalties for each row and column by taking the difference between the smallest cost and next highest cost available in that row/column. If there are two smallest costs, then the penalty is zero.
- ii. Select the row/column, which has the largest penalty and make allocation in the cell having the least cost in the selected row/column. If two or more equal penalties exist, select one where a row/column contains minimum unit cost. If there is again a tie, select one where maximum allocation can be made.
- iii. Delete the row/column, which has satisfied the supply and demand.
- iv. Repeat steps (i) and (ii) until the entire supply and demands are satisfied.
- v. Obtain the initial basic feasible solution.

The initial solution obtained by any of the three methods must satisfy the following conditions:

- a. The solution must be feasible, i.e., the supply and demand constraints must be satisfied (also known as rim conditions).
- b. The number of positive allocations, N must be equal to $m+n-1$, where m is the number of rows and n is the number of columns.

For Example - The transportation problem is given

		Destination				
		D1	D2	D3	D4	Supply(S_i)
Source	O1	3	1	7	4	250
	O2	2	6	5	9	350
	O3	8	3	3	2	400
Demand(D_j):		200	300	350	150	

Solution:

Step 1: Check whether the problem is balanced or not.

If the total sum of all the supply from sources **O1**, **O2**, and **O3** is equal to the total sum of all the demands for destinations **D1**, **D2**, **D3** and **D4** then the transportation problem is a balanced transportation problem.

		Destination				
		D1	D2	D3	D4	Supply(S_i)
Source	O1	3	1	7	4	250
	O2	2	6	5	9	350
	O3	8	3	3	2	400
Demand(D_j):		200	300	350	150	1000

Note: If the problem is not unbalanced then the concept of a dummy row or a dummy column to transform the unbalanced problem to balanced can be followed.

Step 2: Finding the initial basic feasible solution.

Any of the three aforementioned methods can be used to find the initial basic feasible solution. Here, [NorthWest Corner Method](#) will be used. And according to the NorthWest Corner Method this is the final initial basic feasible solution:

		Destination				Supply(S_i)
		D1	D2	D3	D4	
Source	O1	200	50			250
	O2		250	100		350
	O3			250	150	400
Demand(D_j):		200	200	350	150	1000
		0	0	0	0	

Now, the total cost of transportation will be $(200 * 3) + (50 * 1) + (250 * 6) + (100 * 5) + (250 * 3) + (150 * 2) = 3700$.

Step 3: U-V method to optimize the initial basic feasible solution.

The following is the initial basic feasible solution:

200	50		
3	1	7	4
	250	100	
2	6	5	9
8	3	250	150
		3	2

– For U-V method the values u_i and v_j have to be found for the rows and the columns respectively. As there are three rows so three u_i values have to be found i.e. u_1 for the first row, u_2 for the second row and u_3 for the third row.

Similarly, for four columns four v_j values have to be found i.e. v_1 , v_2 , v_3 and v_4 . Check the image below:

	$v_1 =$	$v_2 =$	$v_3 =$	$v_4 =$
$u_1 =$	200	50		
	3	1	7	4
$u_2 =$		250	100	
	2	6	5	9
$u_3 =$			250	150
	8	3	3	2

There is a separate formula to find u_i and v_j ,

$u_i + v_j = C_{ij}$ where C_{ij} is the cost value only for the allocated cell. Read more about it [here](#).

Before applying the above formula we need to check whether $m + n - 1$ is equal to the total number of allocated cells or not where m is the total number of rows and n is the total number of columns.

In this case $m = 3$, $n = 4$ and total number of allocated cells is 6 so $m + n - 1 = 6$. The case when $m + n - 1$ is not equal to the total number of allocated cells will be discussed in the later posts.

Now to find the value for u and v we assign any of the three u or any of the four v as 0. Let we assign $u_1 = 0$ in this case. Then using the above formula we will get $v_1 = 3$ as $u_1 + v_1 = 3$ (i.e. C_{11}) and $v_2 = 1$ as $u_1 + v_2 = 1$ (i.e. C_{12}). Similarly, we have got the value for $v_2 = 1$ so we get the value for $u_2 = 5$ which implies $v_3 = 0$. From the value of $v_3 = 0$ we get $u_3 = 3$ which implies $v_4 = -1$. See the image below:

	$v_1 = -3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$	3	1	7	4
$u_2 = 5$	2	6	5	9
$u_3 = 3$	8	3	3	2

Now, compute penalties using the formula $P_{ij} = u_i + v_j - C_{ij}$ only for unallocated cells. We have two unallocated cells in the first row, two in the second row and two in the third row. Lets compute this one by one.

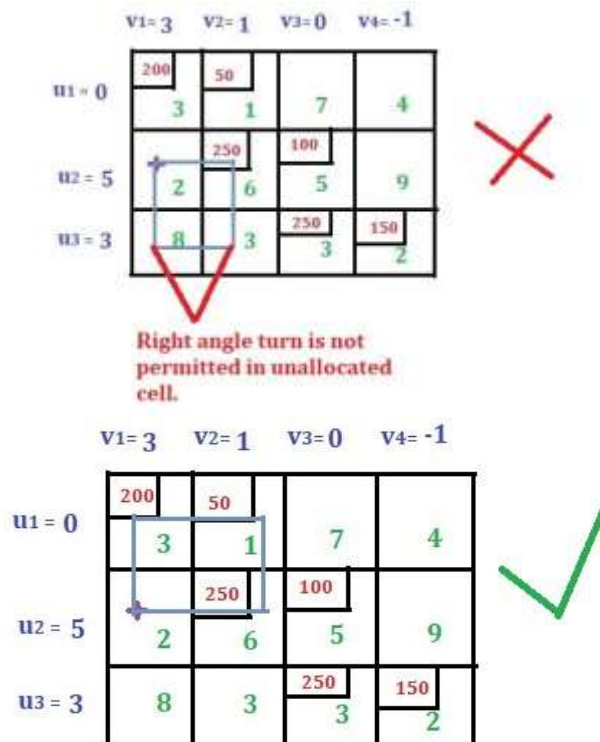
1. For C_{13} , $P_{13} = 0 + 0 - 7 = -7$ (here $C_{13} = 7$, $u_1 = 0$ and $v_3 = 0$)
2. For C_{14} , $P_{14} = 0 + (-1) - 4 = -5$
3. For C_{21} , $P_{21} = 5 + 3 - 2 = 6$
4. For C_{24} , $P_{24} = 5 + (-1) - 9 = -5$
5. For C_{31} , $P_{31} = 3 + 3 - 8 = -2$
6. For C_{32} , $P_{32} = 3 + 1 - 3 = 1$

The Rule: If we get all the penalties value as zero or negative values that mean the optimality is reached and this answer is the final answer. But if we get any positive value means we need to proceed with the sum in the next step.

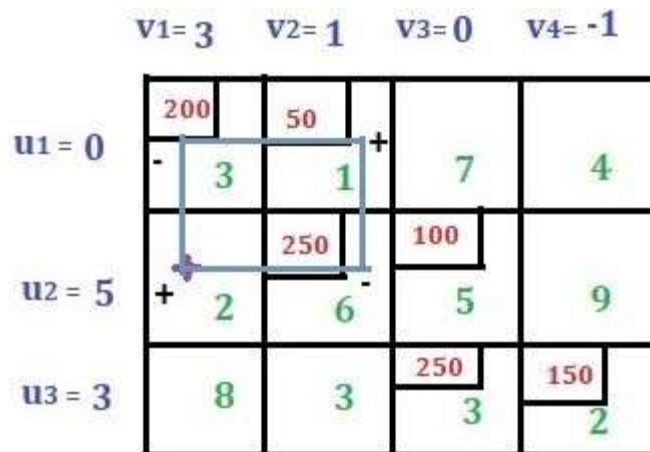
Now find the maximum positive penalty. Here the maximum value is 6 which corresponds to C_{21} cell. Now this cell is new basic cell. This cell will also be included in the solution.

	$v_1 = 3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$	200	50		
	3	1	7	4
$u_2 = 5$	2	6	5	9
$u_3 = 3$	8	3	3	2

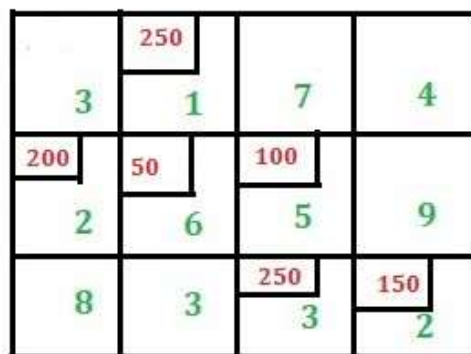
The rule for drawing closed-path or loop. Starting from the new basic cell draw a closed-path in such a way that the right angle turn is done only at the allocated cell or at the new basic cell. See the below images:



Assign alternate plus-minus sign to all the cells with right angle turn (or the corner) in the loop with plus sign assigned at the new basic cell.



Consider the cells with a negative sign. Compare the allocated value (i.e. 200 and 250 in this case) and select the minimum (i.e. select 200 in this case). Now subtract 200 from the cells with a minus sign and add 200 to the cells with a plus sign. And draw a new iteration. The work of the loop is over and the new solution looks as shown below.



Check the total number of allocated cells is equal to $(m + n - 1)$. Again find u values and v values using the formula $u_i + v_j = C_{ij}$ where C_{ij} is the cost value only for allocated cell. Assign $u_1 = 0$ then we get $v_2 = 1$. Similarly, we will get following values for u_i and v_j .

	$v_1 = -3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$		250		
	3	1	7	4
$u_2 = 5$	200	50	100	
	2	6	5	9
$u_3 = 0$	8	3	250	150
			3	2

Find the penalties for all the unallocated cells using the formula $P_{ij} = u_i + v_j - C_{ij}$.

1. For C_{11} , $P_{11} = 0 + (-3) - 3 = -6$
2. For C_{13} , $P_{13} = 0 + 0 - 7 = -7$
3. For C_{14} , $P_{14} = 0 + (-1) - 4 = -5$
4. For C_{24} , $P_{24} = 5 + (-1) - 9 = -5$
5. For C_{31} , $P_{31} = 0 + (-3) - 8 = -11$
6. For C_{32} , $P_{32} = 3 + 1 - 3 = 1$

There is one positive value i.e. 1 for C_{32} . Now this cell becomes new basic cell.

	$v_1 = -3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$		250		
	3	1	7	4
$u_2 = 5$	200	50	100	
	2	6	5	9
$u_3 = 3$	8	3	250	150
		+	3	2

Now draw a loop starting from the new basic cell. Assign alternate plus and minus sign with new basic cell assigned as a plus sign.

	$v_1 = -3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$
$u_1 = 0$		250		
	3	1	7	4
$u_2 = 5$	200	50	100	
	2	-	6	+5
$u_3 = 3$	8	+	3	250
		+	-	3
				150
				2

Select the minimum value from allocated values to the cell with a minus sign. Subtract this value from the cell with a minus sign and add to the cell with a plus sign. Now the solution looks as shown in the image below:

	250		
3	1	7	4
200		150	
2	6	5	9
8	50	200	150
	3	3	2

Check if the total number of allocated cells is equal to $(m + n - 1)$. Find u and v values as above.

	$v_1 = -2$	$v_2 = 1$	$v_3 = 1$	$v_4 = 0$
$u_1 = 0$		250		
	3	1	7	4
$u_2 = 4$	200		150	
	2	6	5	9
$u_3 = 2$	8	50	200	150
		3	3	2

Now again find the penalties for the unallocated cells as above.

1. For $P_{11} = 0 + (-2) - 3 = -5$
2. For $P_{13} = 0 + 1 - 7 = -6$
3. For $P_{14} = 0 + 0 - 4 = -4$
4. For $P_{22} = 4 + 1 - 6 = -1$
5. For $P_{24} = 4 + 0 - 9 = -5$
6. For $P_{31} = 2 + (-2) - 8 = -8$

All the penalty values are negative values. So the optimality is reached.

Now, find the total cost i.e. $(250 * 1) + (200 * 2) + (150 * 5) + (50 * 3) + (200 * 3) + (150 * 2) = 2450$